# Übungsblatt 6

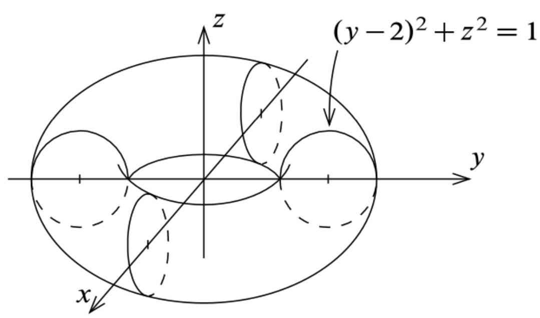
**Exercice 1.** Let $M, N$ be smooth manifolds. Show that $T(M \times N) \approx TM \times TN$.

**Exercice 2.** Show that $T\mathbb{S}^1 \approx \mathbb{R} \times \mathbb{S}^1$.

**Exercice 3.** Consider the smooth map

$$f\colon \mathbb{R}^2 \longrightarrow \mathbb{R}^3, \quad (u,v) \mapsto ((2+\cos(2u))\cos(2v), (2+\cos(2u))\sin(2v), \sin(2u))$$

(1) Show that $f$ is a smooth immersion.
(2) Show that its image is the surface obtained by revolving the circle $\{(y,z) \mid (y-2)^2 + z^2 = 1\}$ in the $(y,z)$-plane about the $z$-axis.




**Exercice 4.** Show that the map $\mathbb{R} \longrightarrow \mathbb{S}^1$, $t \mapsto \exp(2\pi i t)$, is a local diffeomorphism but is not a diffeomorphism.

**Exercice 5.** Show the following properties.

(1) Every composition of local diffeomorphisms is a local diffeomorphism.
(2) Every finite product of local diffeomorphisms between smooth manifolds is a local diffeomorphism.
(3) Every local diffeomorphism is a local homeomorphism and an open map.
(4) The restriction of a local diffeomorphism to an *open* subset is a local diffeomorphism.
(5) Every diffeomorphism is a local diffeomorphism.
(6) Every bijective local diffeomorphism is a diffeomorphism.
(7) A map between smooth manifolds is a local diffeomorphism if and only if, in a neighborhood of each point of its domain, it has a coordinate representation that is a local diffeomorphism.