Übungsblatt 11 (Repetitionsblatt)

<center>To hand in on Monday, 12 May</center>

Exercice 1. What is a smooth manifold?

Exercice 2. Give an example of a continuous manifold that is not a smooth manifold.

Exercice 3. Give a smooth atlas of the torus.

Exercice 4. Let N be a smooth manifold and $\iota \colon M \hookrightarrow N$ an embedded submanifold. Show that for every $p \in M$, there is a natural embedding $T_p M \hookrightarrow T_p N$.

Exercice 5. A *Lie group* G is a smooth manifold that is also a group, and multiplication $m \colon G \times G \longrightarrow G$, $(g, h) \longrightarrow gh$ and inversion $\iota \colon G \longrightarrow G$, $g \mapsto g^{-1}$ are smooth maps of manifolds.

(1) Show that $\mathrm{GL}_n(\mathbb{R})$ is a Lie group for any $n \geqslant 1$.
(2) Show that for any $n \geqslant 1$, the determinant $\det \colon \mathrm{GL}_n(\mathbb{R}) \longrightarrow \mathbb{R}^*$ is smooth homomorphism of groups.
(3) Let G, H be two Lie groups and $f \colon G \longrightarrow H$ a smooth group homomorphism.
  - Show that f has constant rank.
  - Show that if f is bijective, then f^{-1} is smooth.